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ABSTRACT

The purpose of this study was to ascertain whether children in grade 3 who differ in cognitive processing capacity add and subtract differently. The researchers drew upon information from three sources: individual results from a battery of 14 tests, an objective-referenced achievement test measuring a variety of arithmetic skills related to addition and subtraction, and coded strategy data for a set of verbal addition and subtraction problems administered in an interview setting. Eleven children starting third grade in Tasmania, Australia were involved in the study. The data suggested two notions: (1) there is a group of children who have the capacity to reason about quantitative problems, know the basic procedures of addition and subtraction, but see little reason to use those algorithmic procedures to find answers to verbal problems; (2) there is a second group of children whose capacity to reason about quantitative problems is suspect: they do not know the basic procedures to addition and subtraction and have not acquired other skills like modeling or counting which would help them solve verbal problems. It is suggested that educators need to reexamine the relationships between the algorithm and its application. (MNS)

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Cognitive Functioning and Performance on Addition and Subtraction Algorithms

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The basic question addressed in this paper is: Do children in the third grade who differ in cognitive processing capacity add and subtract differently? To examine this question, we have drawn upon information gathered from three different sources to see if in combination they could suggest reasons for anticipated differences. Each source taken separately tends to examine the phenomena under consideration from a microscopic point of view. Our intention was to incorporate the data from the different sources into one picture, since all were derived from the same sample of children, with a view to seeing the phenomena macroscopically.

The sources are: First, individual results from a battery of 14 tests selected to ascertain the cognitive processing capacity of young children. Second, an objective referenced-achievement test measuring a variety of arithmetic skills related to addition and subtraction. And third, coded strategy data for a set of verbal addition and subtraction problems administered in an interview sitting.

The subjects upon which this data were available are 11 children who were starting third grade in Waimea Heights Primary School in Sandy Bay, Tasmania, in February, 1980. Sandy Bay is a middle-class suburb of Hobart, the capital of Australia's smallest state. These children are a subsample of a group of students in an extensive series of cross-sectional studies carried out by the authors in 1979-80.

The Level of Cognitive Functioning

Nearly all researchers in the various areas of mathematics education have indicated the necessity to take heed of the child's intellectual abilities when designing curriculum units and instructional techniques. For this project, a composite strategy was adopted to determine cognitive processing capacity. First, it was necessary to find and use measures of cognitive functioning which appeared logically related to the learning of mathematical material and which seemed to be in tune with the children's level of development. We selected instruments which could be shown prima facie to contain tasks related to early mathematical learning such as number conservation and counting. It was decided to give two batteries of tests. The first battery included four tests designed to measure working memory capacity (M-space) with mathematical type material: in other words, the capacity of the child for processing this kind of material. The second battery included 10 tests constructed to measure the child's level of cognitive development on dimensions familiar from the Piagetian model and presumably related to mathematical ability and as conservation of number and transitivity. These 14 tests were administered to a sample of children (see Romberg & Collis, 1980a, 1980b for details). Second, we used psychometric procedures, factor analysis, and cluster analyses, to interpret the data from both batteries and grouped children. From this approach we decided six well defined sets of children with specific cognitive characteristics had been identified. Without spelling out the details of this grouping, we characterized the groups as follows:

Group 1 children are at M-space Level 1, they performed below the other groups on all tests, they are in general incapable of handling

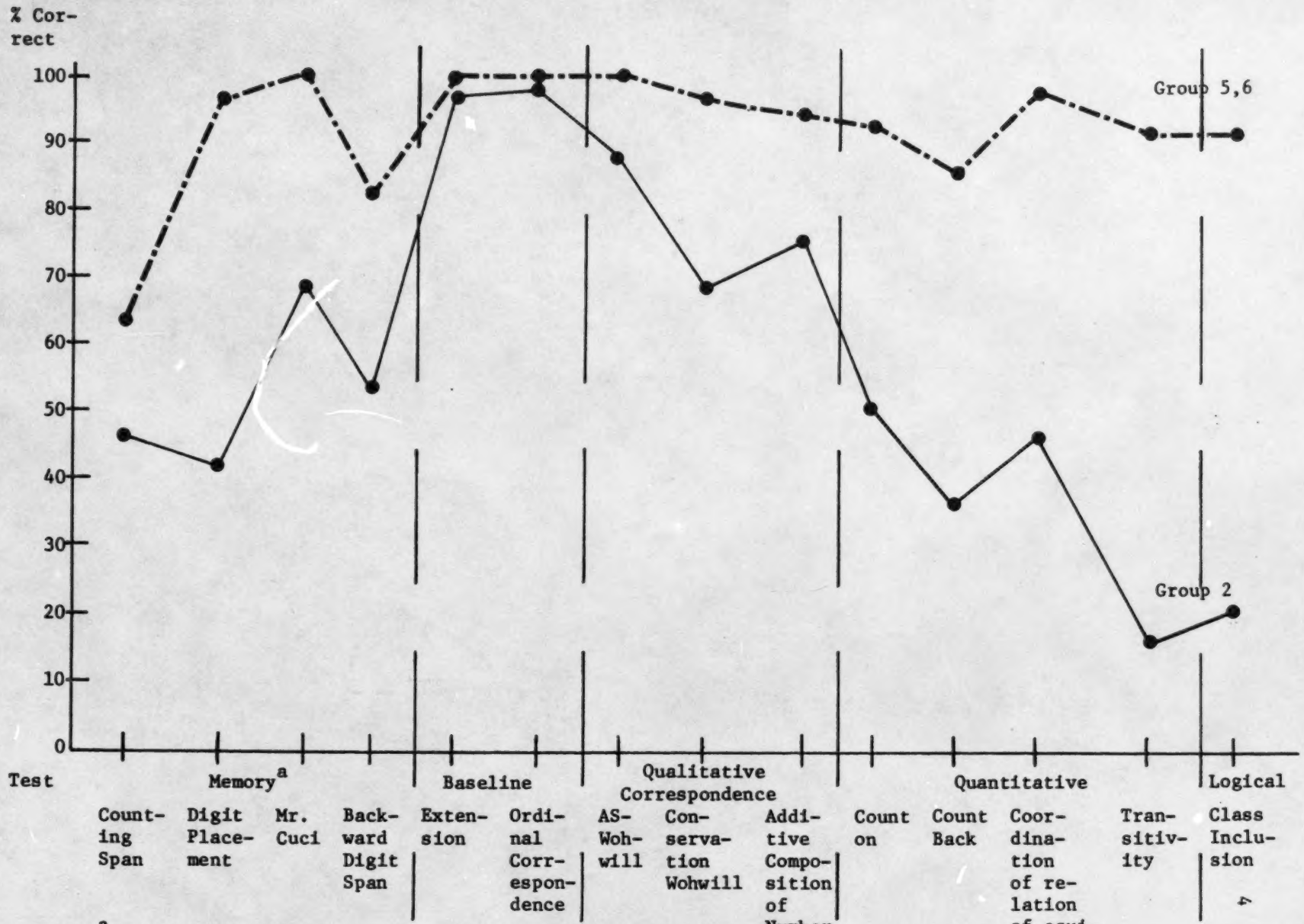
Group 2 children are at M-space Level 2, they are also without specific quantitative and logical skills (although they performed considerably better than Group 1 on all the tests), and they can handle qualitative correspondence at an acceptable level.

Group 3 children are at M-space Level 2S+ (Level 2 on quantitative memory, Level 3 on spatial memory), they are high on qualitative correspondence, they have developed the specific counting skills of counting on and counting back, but they are inadequate in their use of those skills on the transitive reasoning test, and they are inadequate in their use of those skills on the transitive reasoning test, and they are inadequate on logical reasoning.

Group 4 children are at M-space Level 3S- (Level 3 on quantitative memory but Level 2 on spatial memory) are high on qualitative correspondence, they perform well all the quantitative tests, but they are inadequate on the logical reasoning test.

Groups 5 and 6 were combined since both were small and differed only in memory space with M-space Levels 3S+ and 4S-, respectively. Both groups reached the ceiling on the qualitative correspondence tests, have very high scores on all the quantitative tests, and also are high on the logical reasoning test.

For this analysis, we have chosen to contrast 11 children for which other data are available. Four children are members of Group 2 (the lowest level for children in third grade) and seven children are members of Group 5,6. The differences in these children's scores on the 14 tests is shown in Figure 1. Except for the two baseline tests (administered to see if children could respond and follow directions), the differences between the two groups is dramatic. Thus, we conclude these groups of children differ in their



[a]For the Memory tests, M Space #4 is considered 100%.

Figure 1. Profiles for Group 2 and Group 5,6 students on 14 cognitive processing tests.

capacity to reason about quantitative problems.

Achievement

A battery of paper-and-pencil tests had previously been developed to monitor student achievement on addition and subtraction skills at grades 1, 2, and 3 (Buchanan & Romberg, 1982). The items were written to assess the instructional objectives of six experimental topics designed to teach addition and subtraction as well as to measure performance on certain prerequisite objectives and noninstructional objectives (Romberg, Carpenter, & Moser, 1978). Form V of the Battery was administered to the third-grade children in the study in February, 1980 (see Romberg, Collis, & Buchanan, 1982 for details). This test has four parts: a multiple-choice test, a speeded basic facts subtest, a sentence writing free response subtest, and an addition and subtraction algorithms timed subtest. In all, this test includes a wide range of items tapping the variety of skills related to addition and subtraction.

The performance of the two groups of third-grade children on these items is shown in Figure 2. Obviously the cognitive level group 5,6 performed better than the Group 2 children on all objectives. This difference is clear in spite of the fact both groups had common maths instruction in grade 2 and were receiving the same maths instruction in grade 3. In fact, we can conclude from this data that the Group 5,6 know the skills associated with addition and subtraction, while the Group 2 children are struggling.

Strategies

During the same week, the achievement test was administered and each child was interviewed. An interview consisted of six problem types (tasks)

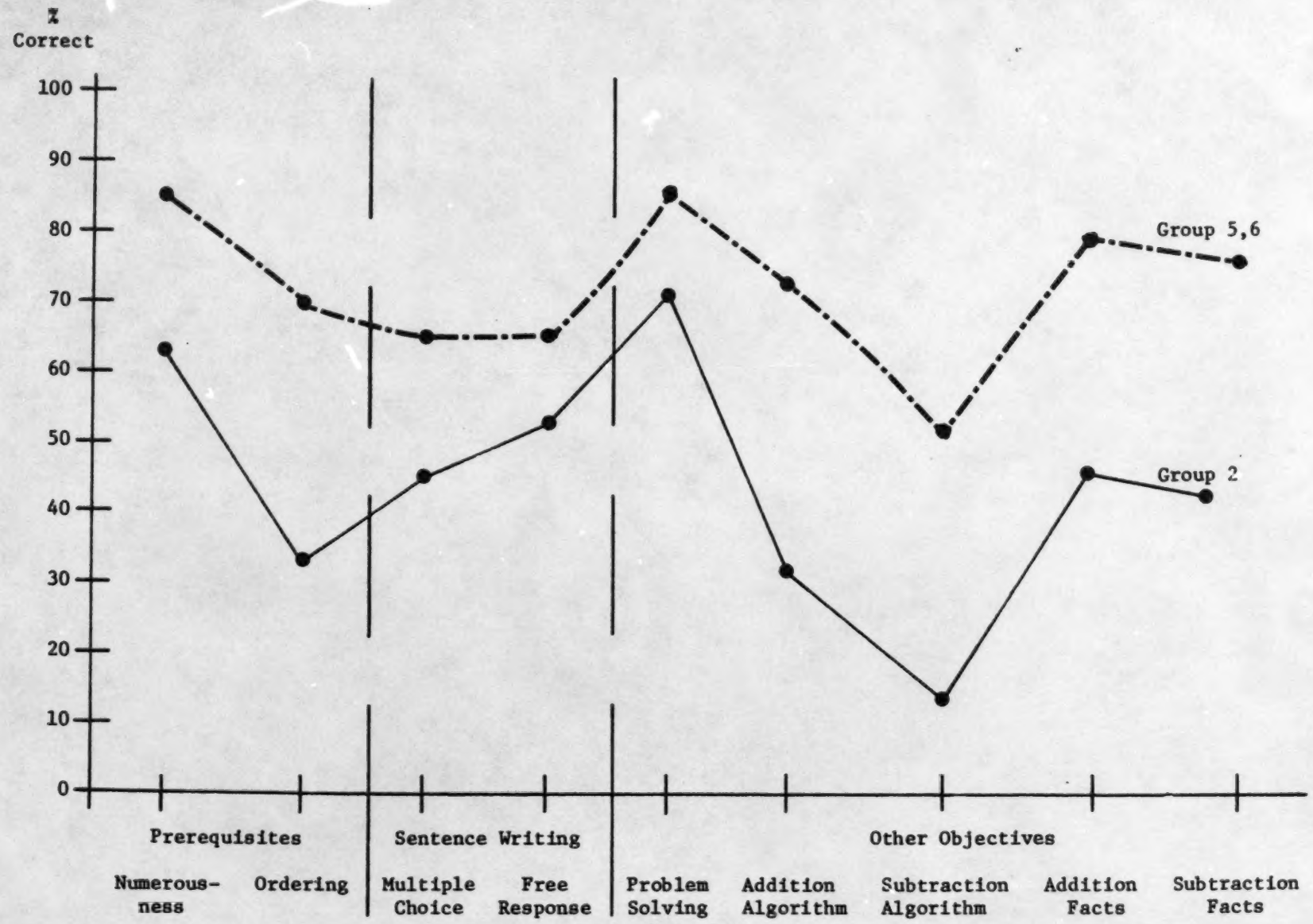


Figure 2. Profiles for Group 2 and Group 5,6 students on achievement objectives.

given under four os six conditions. The six types included two problems solvable by addition of the two given numbers and four problems solvable by subtraction of the two given numbers. The characterization for these six problem types is detailed in Moser (1979) and in Carpenter and Moser (1979).

Table 1 presents representative problems in the order in which the problems were administered to the children. The actual working for each problem type differed in the four conditions, but the semantic structure remained constant.

Within each problem, two of three numbers from a number triple (x, y, z) defined by $x + y = z$, $x < y < z$, were given. In the two addition problems, x, y were presented, with the smaller number x always given first. In the four subtraction problems, z and the larger addend y were presented. The order of presentation of y and z varied among problem types.

The six problem types were presented under six conditions. Only two of which are discussed in this paper. For the interviews with third-grade children, the domain of 2-digit numbers was included. In the 2-digit domain, two subdomains were identified. In the first no regrouping (borrowing or carrying) is required to determine a difference or sum when a computational algorithm is used. In the second subdomain, regrouping is required. The no regrouping set is called the "D" problem set while the regrouping set is referred to as the "E" problems. For the two-digit problems, the sum z is restricted to numbers in the 20s and 30s.

Two trained interviewers (see Martin & Moser, 1980 for details) of interviewer training and reliability) administered the interviews. Each

Table 1

Representative Problem Types

Task 1.	Joining (Addition)	Pam had 3 shells. Her brother gave her 6 more shells. How many shells did Pam have altogether?
Task 2.	Separating (Subtraction)	Jenny had 7 erasers. She gave 5 erasers to Ben. How many erasers did Jenny have left?
Task 3.	Part-Part-Whole Missing addend (Subtraction)	There are 5 fish in a bowl. 3 are striped and the rest are spotted. How many spotted fish are in the bowl?
Task 4.	Part-Part-Whole (Addition)	Matt has 2 baseball cards. He also had 4 football cards. How many cards does Matt have altogether?
Task 5.	Comparison (Subtraction)	Angie has 4 lady bugs. Her brother Todd has 7 lady bugs. How many more lady bugs does Todd have than Angie?
Task 6.	Joining Missing Addend (Subtraction)	Gene has 5 marshmallows. How many more marshmallows does he have to put with them so he has 8 marshmallows altogether?

interviewer was able to conduct 8 to 12 interviews a day. The interviewers were assigned interview areas, which were quiet rooms separate from distracting activities. The verbal tasks were read and reread to the child as often as necessary so that remembering the given number or relationships caused no difficulty. Counting chips, paper, and a pencil were available for children to use. The sessions lasted 15-25 minutes each, with each child receiving the same sequence of problems.

All of the possible codings of student responses are presented in detail in Cookson and Moser (1980). Three or four elements were coded for each child: model used, correctness, strategy, and if incorrect, error. A record of each subject's response to the tasks were compiled from the coding sheets. The data were then summarized in terms of two categories: percent correct and general strategy. The model, strategy, and error data were aggregated into eight independent general strategy categories for the D and E data (non-sentence/direct modeling, non-sentence/counting, non-sentence/routine mental operation, non-sentence/non-routine mental operation, non-sentence/inappropriate, sentence/algorithms, sentence/non-algorithmic, inappropriate sentence). (See Romberg, Collis, & Buchanan, 1981 for details.)

Profiles of the percent correct for the two groups on the D and E tasks is presented in Figure 3. Again and not surprisingly, the differences on each of the 12 tasks are striking. Of more interest, however, is the information on strategies the children used to work these problems. The frequency of use of strategy by these two groups of children is shown in Table 2. This reveals two important facts. First, neither group tends to solve these problems by writing a correct sentence and then using an

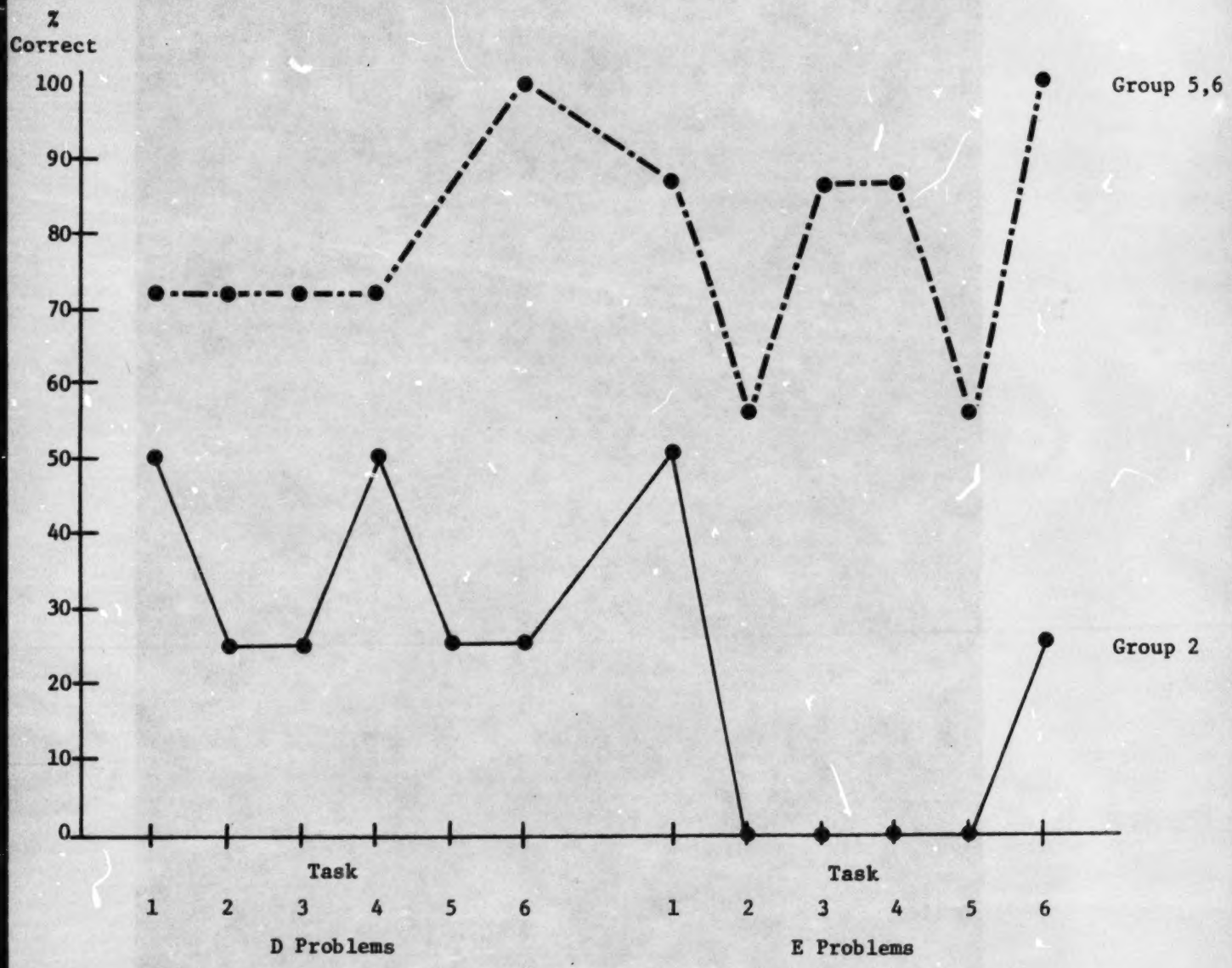


Figure 3. Profiles for Group 2 and Group 5,6 students on interview tasks for D and E problems.

Table 2

Frequency of Use of Strategies by Cognitive Level and Category for All Level D,E Tasks (February, 1980)

Group	No sentence					Correct sentence		Incorrect sentence
	Direct modeling	Counting	Routine mental op.	Nonroutine mental op.	Inapprop.	Alg.	Non-alg.	All strat.
	f/%	f/%	f/%	f/%	f/%	f/%	f/%	f/%
2	3/06	4/08	5/10	3/06	24/50	8/17	1/02	0/0
5,6	32/38	25/30	12/14	2/02	2/02	10/12	0/0	1/01

algorithm to find the answer even though both their second- and third-grade teachers were confident that that would be the strategy used by all children. In fact, the Group 2 children used that strategy more often than the Group 5,6 students. In all instances, the Group 2 children who used an algorithm strategy made an error (either "buggy" or reverse operation) while only one reverse operation error was made by a Group 5,6 child. Second, the actual strategies used by the Group 5,6 were direct modeling (using chips) or counting. In fact, after observing this behavior, some children were asked if they could have found the answer by writing a sentence and using an algorithm. Most indicated they saw no reason to since they knew they could find the answer. Furthermore, it is only on the addition without regrouping tasks that children who are able to work and symbolize problems algorithmically tend to use those algorithms. For addition with regrouping and subtraction, this is not the case. Apparently confidence in use of the algorithms coupled with the verbal problem structure dictates whether a child uses an algorithmic strategy to solve verbal problems.

To see if additional instruction (mostly practice on addition and subtraction algorithms) would change their choice of strategies, we interviewed all children two more times during the third grade. The strategy data from the third interview (May 1980) for these groups is presented in Table 3. The shift is what one would expect in that the "sentence-algorithm" strategy is now more commonly used by both groups. But in neither case is it used on a majority of tasks. The Group 2 children are more likely to use it than the Group 5,6 children, and the increase in use is mostly on subtraction tasks. Furthermore,

Table 3

Frequency of Use of Strategies by Cognitive Level and Category for All Level D,E Tasks (May, 1980)

Group	No sentence					Correct sentence		Incorrect sentence
	Direct modeling f/%	Counting f/%	Routine mental op. f/%	Nonroutine mental op. f/%	Inapprop. f/%	Alg. f/%	Non-alg. f/%	All strat. f/%
2	3/06	6/12	0/0	1/02	15/31	20/42	1/02	2/04
5,6	6/07	27/32	15/18	0/0	3/04	32/38	0/0	1/01

the Group 5,6 children have now given up direct modeling but have not given up counting to solve these tasks.

Conclusions

We think this data suggest two notions about children and their learning to add and subtract.

First, there are a group of children who have the capacity to reason about quantitative problems, know the basic procedures of addition and subtraction, but see little reason to use those algorithmic procedures to find answers to verbal problems. They are confident and comfortable that the procedures they use (direct modeling and counting) are satisfactory. Thus, while educators assume use of algorithms is more efficient, this group of children either fail to see the connection between the semantics of the verbal task and choice of algorithm or fail to be convinced of its efficiency.

Second, there is a second group of children whose capacity to reason about quantitative problems is suspect, they do not know the basic procedures to addition and subtraction and furthermore have not acquired other skills like direct modeling or counting which would help them solve verbal problems. When given verbal problems, they may try to use algorithms (more often than the other group) but not with confidence.

As educators we need to reexamine the relationships between the algorithm and its application. Perhaps current emphasis is misplaced at least at this early stage; perhaps we should treat problem-solving strategies and algorithmic procedures as discrete entities, teach them separately and worry about bringing them together at a later stage in the child's mathematical development.

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